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Magnetized orbifold models

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Abstract: We study (4+2n)-dimensional N=1 super Yang-Mills theory on the orbifold background with non-vanishing magnetic fluxes. In particular, we study zero-modes of spinor fields. The flavor structure of our models is different from one in magnetized torus models, and would be interesting in realistic model building.

Keywords: Field Theories in Higher Dimensions, Supersymmetric Effective Theories, Compactification and String Models, D-branes.

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Contents

1. Introduction

Extra dimensional field theories, in particular string-derived extra dimensional field theories, play an important role in particle physics as well as cosmology. When we start with extra dimensional theories, how to realize chiral theory is one of important issues from the viewpoint of particle physics. Introducing magnetic fluxes in extra dimensions is one way to realize chiral fermions in field theories and superstring theories [1 − 8]. In particular, magnetized D-brane models are T-duals of intersecting D-brane models and several interesting models have been constructed within the framework of intersecting D-brane models [4 − 6, 9 − 11].[1]

The generation number in magnetized extra dimensional models is fixed by the magnetic flux in the same way that the generation number in intersecting D-brane models is fixed by the intersecting number. Yukawa couplings as well as other couplings in four-dimensional effective field theory can be calculated in magnetized extra dimensional models as the overlapping integral of wave functions in the extra dimensional space. We would obtain hierarchically small Yukawa couplings when the overlap integral of wave functions is suppressed, that is, zero-modes are quasi-localized far away from each other in extra

[1] See for a review [12] and references therein.

dimensions. On the other hand, we would obtain Yukawa couplings of $\mathcal{O}(1)$ when the overlap integral is not suppressed. That is an interesting aspect for the purpose to realize hierarchical patterns of quark/lepton mass matrices. Indeed, Yukawa couplings on magnetized torus have been computed in [7] and it has been shown that the results are the same as those in intersecting D-brane models. However, it is still a challenging issue to derive proper generation numbers and realistic quark/lepton masses and mixing angles in magnetized extra dimensional models as well as intersecting D-brane models.

In this paper, we study orbifold models with non-vanishing magnetic fluxes, in particular N=1 super Yang-Mills theory on such a background. Orbifolding the extra dimensions is another way to derive chiral theories [13]. We will show that four-dimensional effective field theories on magnetized orbifolds have a rich structure and they lead to interesting aspects, which do not appear in magnetized torus models. In particular, it will be found that a new type of flavor structures can appear. We also show semi-realistic models on magnetized orbifolds.

Organization of the paper is as follows. In section 2, we study (4+2n)-dimensional N=1 super Yang-Mills theory, whose extra dimensions are torus with non-vanishing magnetic fluxes. We study fermionic and bosonic fields on the magnetized torus and the flavor structure of four-dimensional effective field theories. Most of section 2 is a review. (See e.g. [7].) In section 3, we study the same D-dimensional N=1 super Yang-Mills theory, but the toroidal orbifold background geometry with non-vanishing magnetic fluxes. We study wavefunctions of fermionic and bosonic fields in the compact space, and flavor structure. We will show semi-realistic models. Section 4 is devoted to conclusion and discussion. In appendix, we give two examples of models.

2. Magnetized torus models

2.1 Extra dimensional super Yang-Mills theory

Let us consider $N = 1$ super Yang-Mills theory in $D = 4 + 2n$ dimensions. Its Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\left(F^{MN}F_{MN}\right) + \frac{i}{2g^2}\mathrm{Tr}\left(\bar{\lambda}\Gamma^M D_M\lambda\right), \tag{2.1}$$

where $M, N = 0, \ldots, (D-1)$. Here, λ denotes gaugino fields, Γ^M is the gamma matrix for D dimensions and the covariant derivative D_M is given as

$$D_M\lambda = \partial_M\lambda - i[A_M, \lambda], \tag{2.2}$$

where A_M is the vector field. Furthermore, the field strength F_{MN} is given by

$$F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]. \tag{2.3}$$

We consider the torus $(T^2)^n$ as the extra dimensional compact space, whose coordinates are denoted by y_m ($m = 4, \ldots, 2n+3$), while the coordinates of four-dimensional uncompact space $R^{3,1}$ are denoted by x_μ ($\mu = 0, \ldots, 3$). We use orthogonal coordinates and choose the torus metric such that y_m is identified by $y_m + n_m$ with $n_m =$ integer. The gaugino fields λ and the vector fields A_m corresponding to the compact directions are decomposed as

$$\lambda(x,y) = \sum_n \chi_n(x) \otimes \psi_n(y), \tag{2.4}$$

$$A_m(x,y) = \sum_n \varphi_{n,m}(x) \otimes \phi_{n,m}(y). \tag{2.5}$$

2.2 U(1) gauge theory on magnetized torus T^2

First, let us consider U(1) gauge theory on T^2 with the coordinates (y_4, y_5). We study the non-vanishing constant magnetic flux $F_{45} = 2\pi M$. We use the following gauge,

$$A_4 = 0, \qquad A_5 = 2\pi M y_4. \tag{2.6}$$

Then, their boundary conditions can be written as

$$\begin{aligned} A_m(y_4+1, y_5) &= A_m(y_4, y_5) + \partial_m \chi_4, \qquad \chi_4 = 2\pi M y_5, \\ A_m(y_4, y_5+1) &= A_m(y_4, y_5) + \partial_m \chi_5, \qquad \chi_5 = 0. \end{aligned} \tag{2.7}$$

Now, we study the spinor field $\psi(y)$ with the U(1) charge $q = \pm 1$ on T^2, which corresponds to the compact part in the decomposition (2.4). The zero-mode satisfies the following equation,

$$\tilde{\Gamma}^m (\partial_m - iqA_m)\psi(y) = 0, \tag{2.8}$$

for $m = 4, 5$, where $\tilde{\Gamma}^m$ corresponds to the gamma matrix for the two-dimensional torus T^2, e.g.

$$\tilde{\Gamma}^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tilde{\Gamma}^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{2.9}$$

and $\psi(y)$ is the two component spinor,

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}. \tag{2.10}$$

Because of (2.7), the spinor field satisfies the following boundary condition,

$$\psi(y_4+1, y_5) = e^{iq\chi_4}\psi(y_4, y_5) = e^{2\pi i q M y_5}\psi(y_4, y_5), \tag{2.11}$$

$$\psi(y_4, y_5+1) = e^{iq\chi_5}\psi(y_4, y_5) = \psi(y_4, y_5). \tag{2.12}$$

The consistency for the contractible loop, i.e. $(y_4, y_5) \to (y_4+1, y_5) \to (y_4+1, y_5+1)$, requires $M =$ integer. Because of the periodicity along y_5, $\psi_\pm$ can be written by

$$\psi_\pm(y_4, y_5) = \sum_n c_{\pm,n}(y_4) e^{2\pi i n y_5}. \tag{2.13}$$

Suppose that $qM > 0$. Then, the solution for the zero-mode equation of ψ_+ is given by

$$c_{+,n}(y_4) = k_{+,n} e^{-\pi q M y_4^2 + 2\pi n y_4}, \tag{2.14}$$

where $k_{+,n}$ is a constant. Furthermore the boundary condition requires

$$k_{+,n} = a_n e^{-\pi n^2/(qM)}, \tag{2.15}$$

and a_{n+qM} is equal to a_n, i.e. $a_{n+qM} = a_n$. Thus, there are $|M|$ independent zero modes of ψ_+, which have normalizable wave functions,

$$\Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)\,, \tag{2.16}$$

for $j = 0, \dots, M-1$, where N_j is a normalization constant and

$$\vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi) = \sum_n e^{-M\pi(n+j/M)^2 + 2\pi(n+j/M)M(y_4+iy_5)}, \tag{2.17}$$

that is, the Jacobi theta-function. We can introduce the complex structure modulus τ by replacing the above Jacobi theta-function as

$$\vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi) \to \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + \tau y_5), M\tau)\,. \tag{2.18}$$

Thus, zero-mode wave functions depend on only the complex structure modulus, but not the overall size of T^2. Furthermore, there is the degree of freedom to shift $y_m \to y_m + d_m$ with constants d_m. They correspond to constant Wilson lines.

On the other hand, the zero-modes equation for ψ_- can be solved in a similar way, but their wave functions are unnormalizable. Hence, we can derive chiral theory by introducing magnetic fluxes. When $qM < 0$, ψ_- has $|M|$ independent zero modes with normalizable wave functions, while zero modes for ψ_+ have unnormalizable wave functions. Bosonic fields are analyzed in a similar way. (See e.g. [7].)

2.3 U(N) gauge theory on magnetized torus T^2

Here, we study U(N) gauge theory on T^2. Let us consider the following form of (abelian) magnetic flux

$$F_{45} = 2\pi \begin{pmatrix} M_1 \mathbf{1}_{N_1\times N_1} & & 0 \\ & \ddots & \\ 0 & & M_n \mathbf{1}_{N_n\times N_n} \end{pmatrix}, \tag{2.19}$$

where $\mathbf{1}_{N_a\times N_a}$ denotes $(N_a \times N_a)$ identity matrix. This abelian magnetic flux breaks the gauge group as $\mathrm{U}(N) \to \prod_{a=1}^n \mathrm{U}(N_a)$ with $N = \sum_a N_a$. The rank is not reduced by the abelian magnetic flux. When we consider non-abelian magnetic flux, i.e. the toron background [14], the rank can be reduced.[2] However, here we restrict ourselves to the abelian flux.

[2] See e.g. [15, 16] and references therein.

Now, let us study gaugino fields on this background. We focus on the block including only $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$ and such a block has the following magnetic flux,

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}. \tag{2.20}$$

We use the same gauge as (2.6), i.e.

$$A_4 = 0, \qquad A_5 = F_{45} y_4. \tag{2.21}$$

Similarly, the gaugino fields λ in $R^{3,1} \times T^2$ are decomposed as

$$\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}. \tag{2.22}$$

Furthermore these gaugino fields are decomposed as (2.4),

$$\begin{aligned} \lambda^{aa}(x,y) &= \sum_n \chi_n^{aa}(x) \otimes \psi_n^{aa}(y), & \lambda^{ab}(x,y) &= \sum_n \chi_n^{ab}(x) \otimes \psi_n^{ab}(y), \\ \lambda^{ba}(x,y) &= \sum_n \chi_n^{ba}(x) \otimes \psi_n^{ba}(y), & \lambda^{bb}(x,y) &= \sum_n \chi_n^{bb}(x) \otimes \psi_n^{bb}(y). \end{aligned} \tag{2.23}$$

Each of ψ^{aa}, ψ^{ab}, ψ^{ba} and ψ^{bb} is a two-component spinor $(\psi_+, \psi_-)^T$. Their zero-modes satisfy

$$\begin{pmatrix} \bar{\partial}\psi_+^{aa} & [\bar{\partial} + 2\pi(M_a - M_b)y_4]\psi_+^{ab} \\ [\bar{\partial} + 2\pi(M_b - M_a)y_4]\psi_+^{ba} & \bar{\partial}\psi_+^{bb} \end{pmatrix} = 0, \tag{2.24}$$

$$\begin{pmatrix} \partial\psi_-^{aa} & [\partial - 2\pi(M_a - M_b)y_4]\psi_-^{ab} \\ [\partial - 2\pi(M_b - M_a)y_4]\psi_-^{ba} & \partial\psi_-^{bb} \end{pmatrix} = 0, \tag{2.25}$$

where $\bar{\partial} = \partial_4 + i\partial_5$ and $\partial = \partial_4 - i\partial_5$.

The zero-modes of ψ^{aa} and ψ^{bb} correspond to four-dimensional massless gauginos for the unbroken gauge group $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$. Dirac equations of $\psi^{aa}(y)$ and $\psi^{bb}(y)$ in (2.24) and (2.25) do not include any magnetic fluxes. That is, both of $\psi_\pm$ have the same zero-modes as those on T^2 without magnetic fluxes.

Next, we study spinor fields, λ^{ab} and λ^{ba}, which correspond to bi-fundamental matter fields, $(N_a, \bar{N}_b)$ and $(\bar{N}_a, N_b)$ for the unbroken gauge group $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$. When $M_a - M_b > 0$, λ_+^{ab} and λ_-^{ba} have $(M_a - M_b)$ zero-modes with normalizable wave functions, i.e. $\Theta^j(y_4, y_5)$ for $j = 0, \ldots, (M_a - M_b - 1)$ as (2.16), but zero-mode wave functions of λ_-^{ab} and λ_+^{ba} are unnormalizable. On the other hand, when $M_a - M_b < 0$, λ_-^{ab} and λ_+^{ba} have $(M_b - M_a)$ normalizable zero-modes. Hence, we obtain chiral theory. We have the degree of freedom of the constant shift $y_m \to y_m + d_m$.

Similarly, we can analyze bosonic fields A_m. In general, introduction of non-vanishing magnetic fluxes on T^2 breaks supersymmetry completely.

JHEP09(2008)043

2.4 $\mathrm{U}(N)$ gauge theory on $(T^2)^3$

Here, we extend the previous analysis to $\mathrm{U}(N)$ gauge theory on $(T^2)^3$. We consider the magnetic background, where only F_{45}, F_{67} and F_{89} are non-vanishing, but the others of F_{mn} are vanishing. Furthermore, F_{45}, F_{67} and F_{89} are given by

$$\begin{aligned} F_{45} &= 2\pi \begin{pmatrix} M_1^{(1)}\mathbf{1}_{N_1\times N_1} & & 0 \\ & \ddots & \\ 0 & & M_n^{(1)}\mathbf{1}_{N_n\times N_n} \end{pmatrix}, \\ F_{67} &= 2\pi \begin{pmatrix} M_1^{(2)}\mathbf{1}_{N_1\times N_1} & & 0 \\ & \ddots & \\ 0 & & M_n^{(2)}\mathbf{1}_{N_n\times N_n} \end{pmatrix}, \\ F_{89} &= 2\pi \begin{pmatrix} M_1^{(3)}\mathbf{1}_{N_1\times N_1} & & 0 \\ & \ddots & \\ 0 & & M_n^{(3)}\mathbf{1}_{N_n\times N_n} \end{pmatrix}. \end{aligned} \tag{2.26}$$

This background breaks the gauge group $\mathrm{U}(N)$ as $\mathrm{U}(N) \to \prod_{a=1}^n \mathrm{U}(N_a)$ with $N = \sum_a N_a$.

We can study gaugino fields on this background as a simple extension of the previous section 2.3. That is, we focus on the block including only $\mathrm{U}(N_a)\times\mathrm{U}(N_b)$ and such a block has the following magnetic flux as (2.20),

$$F_{2i+2,2i+3} = 2\pi \begin{pmatrix} M_a^{(i)}\mathbf{1}_{N_a\times N_a} & 0 \\ 0 & M_b^{(i)}\mathbf{1}_{N_b\times N_b} \end{pmatrix}, \tag{2.27}$$

and we use the following gauge

$$A_{2i+2} = 0, \qquad A_{2i+3} = y_{2i+2}F_{2i+2,2i+3}, \tag{2.28}$$

for $i = 1,2,3$. Then, we decompose the gaugino fields $\lambda(x,y)$ as (2.4), i.e. the four-dimensional part $\chi(x)$ and the i-th T^2 part $\psi_{(i)}(y_{2i+2}, y_{2i+3})$, whose zero modes satisfy

$$\begin{aligned} &\begin{pmatrix} \bar\partial_i \psi^{aa}_{(i)+} & [\bar\partial_i + 2\pi(M_a^{(i)} - M_b^{(i)})y_{2i+2}]\psi^{ab}_{(i)+} \\ [\bar\partial_i + 2\pi(M_b^{(i)} - M_a^{(i)})y_{2i+2}]\psi^{ba}_{(i)+} & \bar\partial_i\psi^{bb}_{(i)+} \end{pmatrix} = 0, \\ &\begin{pmatrix} \partial_i \psi^{aa}_{(i)-} & [\partial_i - 2\pi(M_a^{(i)} - M_b^{(i)})y_{2i+2}]\psi^{ab}_{(i)-} \\ [\partial_i - 2\pi(M_b^{(i)} - M_a^{(i)})y_{2i+2}]\psi^{ba}_{(i)-} & \partial_i\psi^{bb}_{(i)-} \end{pmatrix} = 0, \end{aligned} \tag{2.29}$$

where $\bar\partial_i = \partial_{2i+2} + i\partial_{2i+3}$ and $\partial_i = \partial_{2i+2} - i\partial_{2i+3}$.

The gaugino fields, ψ^{aa} and ψ^{bb}, for the unbroken gauge symmetry have no effect from magnetic fluxes in their Dirac equations. Hence, they have the same zero-modes as those on $(T^2)^3$ without magnetic fluxes. On the other hand, ψ^{ab} and ψ^{ba} correspond to bi-fundamental matter fields, $(N_a, \bar N_b)$ and $(\bar N_a, N_b)$. For the i-th T^2 with

$M_a^{(i)} - M_b^{(i)} > 0$, $\psi^{ab}_{(i)+}$ and $\psi^{ba}_{(i)-}$ have $|M_a^{(i)} - M_b^{(i)}|$ normalizable zero-modes, while $\psi^{ab}_{(i)-}$ and $\psi^{ba}_{(i)+}$ have no normalizable zero-modes. When $M_a^{(i)} - M_b^{(i)} < 0$, $\psi^{ab}_{(i)-}$ and $\psi^{ba}_{(i)+}$ have $|M_a^{(i)} - M_b^{(i)}|$ normalizable zero-modes. Then, the total number of bi-fundamental zero-modes is given by $\prod_{i=1}^3 |M_a^{(i)} - M_b^{(i)}|$ and all of them have the same six-dimensional chirality sign $\left[\prod_{i=1}^3 (M_a^{(i)} - M_b^{(i)})\right]$. Since the ten-dimensional chirality of gaugino fields is fixed, bi-fundamental zero-modes for either $(N_a, \bar{N}_b)$ or $(\bar{N}_a, N_b)$ appear with a fixed four-dimensional chirality. To summarize, the total number of bi-fundamental zero-modes for $(N_a, \bar{N}_b)$ is equal to

$$I_{ab} = \prod_{i=1}^{3} (M_a^{(i)} - M_b^{(i)}), \tag{2.30}$$

and their wave functions are given by a product of two-dimensional parts, i.e.

$$\Theta^{i_1,i_2,i_3}(y) = \Theta^{i_1}(y_4, y_5)\Theta^{i_2}(y_6, y_7)\Theta^{i_3}(y_8, y_9), \tag{2.31}$$

for $i_1 = 0, \ldots, (M_a^{(1)} - M_b^{(1)} - 1)$, $i_2 = 0, \ldots, (M_a^{(2)} - M_b^{(2)} - 1)$ and $i_3 = 0, \ldots, (M_a^{(3)} - M_b^{(3)} - 1)$. For $I_{ab} < 0$, this means that there appear $|I_{ab}|$ independent zero modes for $(\bar{N}_a, N_b)$. It is also convenient to introduce the notation, $I^i_{ab} \equiv M_a^{(i)} - M_b^{(i)}$.

Similarly, we can analyze bosonic fields corresponding to A_m for $m = 4, \ldots, 9$. For generic values of magnetic fluxes, supersymmetry is broken completely. However, when they satisfy the following condition [8, 7],

$$\sum_{i=1}^{3} \pm \frac{M_a^{(i)} - M_b^{(i)}}{\mathcal{A}^{(i)}} = 0, \tag{2.32}$$

for one combination of signs, where $\mathcal{A}^{(i)}$ denotes the area of the i-th torus, there appear massless scalar modes as well as massive modes and four-dimensional N=1 supersymmetry remains unbroken at least in the $a - b$ sector. When we consider $\mathcal{A}^{(i)}$ as free parameters, we can realize the above supersymmtric condition (2.32) for most cases by choosing proper values of $\mathcal{A}^{(i)}$. For the case with the universal area, $\mathcal{A}^{(1)} = \mathcal{A}^{(2)} = \mathcal{A}^{(3)}$, the above condition (2.32) reduces to

$$\sum_{i=1}^{3} \pm (M_a^{(i)} - M_b^{(i)}) = 0. \tag{2.33}$$

In addition to (2.32), when one of them is vanishing, i.e. $(M_a^{(i)} - M_b^{(i)}) = 0$ and

$$\sum_{j \neq i} \pm \frac{M_a^{(j)} - M_b^{(j)}}{\mathcal{A}^{(j)}} = 0, \tag{2.34}$$

four-dimensional N=2 supersymmetry is unbroken. In these supersymmetric models, zero-mode profiles of bosonic fields are the same as their superpartners, that is, zero-mode profiles of fermionic fields.

JHEP09(2008)043

2.5 $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$ models with three families

Here, we consider an illustrating model with the unbroken gauge group $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$, which is derived from ten-dimensional $\mathrm{U}(N)$ super Yang-Mills theory on $R^{3,1} \times (T^2)^3$ with the magnetic fluxes as in the previous section. We assume that the magnetic fluxes satisfy the supersymmetric condition (2.32) and massless scalar fields appear as partners of massless spinor fields with bi-fundamental representations. In addition to supersymmetric vector multiplets for the gauge group $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$, the massless spectrum of this model includes three types of bi-fundamental matter fields, $(N_a, \bar{N}_b)$, $(N_b, \bar{N}_c)$ and $(N_c, \bar{N}_a)$. This class of models include the $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ Pati-Salam model for $N_a = 4, N_b = 2$ and $N_c = 2$.[3] In this case, bi-fundamental matter fields $(4,2,1)$, $(\bar{4},1,2)$ and $(1,2,2)$ under $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ include left-handed quarks and leptons in $(4,2,1)$, the up- and down-sectors of right-handed quarks and right-handed charged leptons and neutrinos in $(\bar{4},1,2)$ and up- and down-sectors of Higgs fields in $(1,2,2)$. Indeed, in intersecting D-brane models it is a convenient way that first one constructs the supersymmetric Pati-Salam model and then breaks it to the minimal supersymmetric standard model (MSSM) in order to realize the MSSM-like models within the framework of intersecting D-brane models. (See e.g. [11, 18] and references therein.)[4] For the purpose to derive a semi-realistic model, we consider the realization of three families of $(N_a, \bar{N}_b)$ and $(\bar{N}_a, N_c)$ matter fields, i.e. $I_{ab} = I_{ca} = 3$. Yukawa couplings among $(N_a, \bar{N}_b)$ and $(\bar{N}_a, N_c)$ matter fields and $(N_b, \bar{N}_c)$ Higgs fields in four-dimensional effective theory are given by the overlap integral of zero-mode wave functions (2.31) in extra dimensions [17],

$$Y^{ijk} = g \int dy \; \Theta^{i_1,i_2,i_3}(y) \cdot \Theta^{j_1,j_2,j_3}(y) \cdot \Theta^{k_1,k_2,k_3}(y), \tag{2.35}$$

in the canonically normalized basis.

Now, let us study $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$ models, which lead to three families of $(N_a, \bar{N}_b)$ and $(\bar{N}_a, N_c)$, i.e. $I_{ab} = \prod_{i=1}^3 I^i_{ab} = 3$ and $I_{ca} = \prod_{i=1}^3 I^i_{ca} = 3$. First, it is straightforward to show that we can not derive three-family models, which satisfy the condition (2.33), that is, it is difficult to realize supersymmetric three-family models from ten-dimensional super Yang-Mills theory with the universal area of tori. Thus, we study models with the condition (2.32).

For generic model building with the condition (2.32), we can construct three-family models. Magnetic fluxes leading to three families are classified into two classes. One class corresponds to the following magnetic fluxes,

$$(|I^{(1)}_{ab}|, |I^{(2)}_{ab}|, |I^{(3)}_{ab}|) = (3,1,1), \qquad (|I^{(1)}_{ca}|, |I^{(2)}_{ca}|, |I^{(3)}_{ca}|) = (1,3,1), \tag{2.36}$$

and their permutations, and the other corresponds to

$$(|I^{(1)}_{ab}|, |I^{(2)}_{ab}|, |I^{(3)}_{ab}|) = (3,1,1), \qquad (|I^{(1)}_{ca}|, |I^{(2)}_{ca}|, |I^{(3)}_{ca}|) = (3,1,1), \tag{2.37}$$

[3] $\mathrm{U}(1)^3$ would be anomalous and their gauge bosons would become massive through the Green-Schwarz mechanism.

[4] See e.g. for the Pati-Salam model from heterotic orbifold models [19], where $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is broken to the standard gauge group by vacuum expectation values of scalar fields, $(4,1,2)$ and $(\bar{4},1,2)$, while in the intersecting D-brane models $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is broken by splitting D-branes, that is, vacuum expectation values of adjoint scalar fields.

and their permutations. Hence, we can realize the restricted flavor structure. Moreover, the number of Higgs fields are constrained because $|I_{bc}^{(i)}| = |\pm I_{ab}^{(i)} \pm I_{ca}^{(i)}|$.[5] For example, in the first class of models (2.36) we would obtain

$$|I_{bc}^{(1)}| = 4 \ \text{ or } \ 2, \qquad |I_{bc}^{(2)}| = 4 \ \text{ or } \ 2, \qquad |I_{bc}^{(3)}| = 2 \ \text{ or } \ 0, \tag{2.38}$$

and the total Higgs number would be equal to $\prod_i |I_{bc}^{(i)}| = 0, 8, 16, 32$. On the other hand, in the second class of models (2.37), we would obtain

$$|I_{bc}^{(1)}| = 6 \ \text{ or } \ 0, \qquad |I_{bc}^{(2)}| = 2 \ \text{ or } \ 0, \qquad |I_{bc}^{(3)}| = 2 \ \text{ or } \ 0, \tag{2.39}$$

and the total Higgs number would be equal to $\prod_i |I_{bc}^{(i)}| = 0, 24$. Thus, the total Higgs number would be quite large except the models without Higgs fields. Therefore, for phenomenological applications, it would be important to make the flavor structure richer. That is the purpose of the next section including model building with smaller number of Higgs fields.

3. Magnetized orbifold models

3.1 U(1) gauge theory on magnetized orbifold T^2/Z_2

Now, let us study U(1) gauge theory on the orbifold T^2/Z_2 with the coordinates (y_4, y_5), which are transformed as

$$y_4 \to -y_4, \qquad y_5 \to -y_5, \tag{3.1}$$

under the Z_2 orbifold twist. Then, we introduce the same magnetic flux $F_{45} = 2\pi M$ as one in section 2.2 and use the same gauge as (2.6). Note that this magnetic flux is invariant under the Z_2 orbifold twist.

We study the spinor field $\psi(y)$ on the above background. The spinor field $\psi(y)$ with the U(1) charge $q = \pm 1$ satisfies the same equation as one on T^2, i.e. (2.8). Then, we require $\psi(y)$ transform under the Z_2 twist as

$$\psi(-y_4, -y_5) = (-i)\tilde{\Gamma}^4\tilde{\Gamma}^5 P \psi(-y_4, -y_5), \tag{3.2}$$

where P depends on the charge q like $P = (-1)^{q+n}$ with $n =$ integer and it should satisfy $P^2 = 1$. Suppose that $qM > 0$. Then, there are M independent zero-modes for ψ when we do not take into account the Z_2 projection. However, some of them are projected out by the above Z_2 boundary condition. For example, for $(-i)\tilde{\Gamma}^4\tilde{\Gamma}^5 P = 1$, only even functions remain, while only odd functions remain for $(-i)\tilde{\Gamma}^4\tilde{\Gamma}^5 P = -1$. Note that

$$\Theta^j(-y_4, -y_5) = \Theta^{M-j}(y_4, y_5), \tag{3.3}$$

where $\Theta^M(y_4, y_5) = \Theta^0(y_4, y_5)$. That is, even and odd functions are given by

$$\Theta^j_{\text{even}} = \frac{1}{\sqrt{2}}(\Theta^j + \Theta^{M-j}), \tag{3.4}$$

$$\Theta^j_{\text{odd}} = \frac{1}{\sqrt{2}}(\Theta^j - \Theta^{M-j}), \tag{3.5}$$

[5] Similar results have been obtained in intersecting D-brane models [20, 21], which are T-duals of magnetized D-brane models.

M	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4

Table 1: The numbers of zero-modes with even and odd wave functions.

respectively. Hence, for $M = 2k$ with $k =$ integer and $k > 0$, the number of zero-modes ψ_+ for $P = 1$ and $P = -1$ are equal to $k + 1$ and $k - 1$, respectively. On the other hand, for $M = 2k + 1$ with $k =$ integer and $k \geq 0$, the number of zero-modes ψ_+ for $P = 1$ and $P = -1$ are equal to $k + 1$ and k, respectively. It is interesting that odd functions can correspond to zero-modes in magnetized orbifold models. On the orbifold with vanishing magnetic flux $M = 0$, odd modes correspond to not zero-modes, but massive modes. However, odd modes, which would correspond to massive modes for $M = 0$, mix to lead to zero-modes in the case with $M \neq 0$. It would be convenient to write these results explicitly for later discussions. Table 1 shows the numbers of zero-modes with even and odd wave functions for $M \leq 10$. Note that the degree of constant shift $y_m \to y_m + d_m$, which we have on the torus, is ruled out on the orbifold.

3.2 U(N) gauge theory on magnetized orbifold T^2/Z_2

Now, let us study U(N) gauge theory on the orbifold T^2/Z_2. We consider the same magnetic flux as (2.19), which breaks the gauge group $\mathrm{U}(N) \to \prod_{a=1}^{n} \mathrm{U}(N_a)$. Furthermore, we associate the Z_2 twist with the Z_2 action in the gauge space as

$$A_\mu(x, -y) = PA_\mu(x, y)P^{-1}, \qquad A_m(x, y) = -PA_m(x, y)P^{-1}. \tag{3.6}$$

In general, the Z_2 boundary condition breaks the gauge group $\prod_{a=1}^{n} \mathrm{U}(N_a)$ further. For simplicity, here we restrict ourselves to the Z_2 action, which remains the gauge group $\prod_{a=1}^{n} \mathrm{U}(N_a)$ unbroken. Thus, the Z_2 action is trivial for the unbroken gauge group, but it is not trivial for spinor fields as well as scalar fields.

Here, let us study spinor fields. We focus on the $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$ block (2.20) and use the same gauge as (2.6), i.e. $A_4 = 0$ and $A_5 = F_{45} y_4$. We consider the spinor fields, $\lambda^{aa}_\pm$, $\lambda^{ab}_\pm$, $\lambda^{ba}_\pm$ and $\lambda^{bb}_\pm$, where $\pm$ denotes the chirality in the extra dimension like (2.10). Their Z_2 boundary conditions are given by

$$\lambda_\pm(x, -y) = \pm P \lambda_\pm(x, y) P^{-1}, \tag{3.7}$$

for $\lambda^{aa}_\pm$, $\lambda^{ab}_\pm$, $\lambda^{ba}_\pm$ and $\lambda^{bb}_\pm$. First, we study the gaugino fields, $\lambda^{aa}_\pm$ and $\lambda^{bb}_\pm$ for the unbroken gauge group. Since the Z_2 action P is trivial for the unbroken gauge indices, the above Z_2 boundary conditions reduce to $\lambda^{aa}_\pm(x, -y) = \pm\lambda^{aa}_\pm(x, y)$ and $\lambda^{bb}_\pm(x, -y) = \pm\lambda^{bb}_\pm(x, y)$. In addition, the magnetic flux does not appear in their zero-mode equations. Thus, $\lambda^{aa}_+(x, y)$ as well as $\lambda^{bb}_+(x, y)$ has a zero-mode, but $\lambda^{aa}_-(x, y)$ and $\lambda^{bb}_-(x, y)$ are projected out by the Z_2 orbifold projection as the usual Z_2 orbifold without the magnetic flux.

Next, let us study the bi-fundamental matter fields $\lambda^{ab}_\pm$ and $\lambda^{ba}_\pm$. The magnetic flux $M_a - M_b$ appears in their zero-mode equations. Without the Z_2 projection, there are

$|M_a - M_b|$ zero modes. For example, when $M_a - M_b > 0$, λ_+^{ab} as well as λ_-^{ba} has $(M_a - M_b)$ zero modes with the wave functions Θ^j for $j = 0, \ldots, (M_a - M_b - 1)$. When we consider the Z_2 projection, either even or odd modes remain. For example, when we consider the projection P such that $\lambda_+^{ab}(x, -y) = \lambda_+^{ab}(x, y)$, only zero-modes corresponding to Θ^j_{even} remain and the number of zero-modes is equal to $(M_a - M_b)/2 + 1$ for $(M_a - M_b) =$ even and $(M_a - M_b + 1)/2$ for $(M_a - M_b) =$ odd. On the other hand, when we consider the projection P such that $\lambda_+^{ab}(x, -y) = -\lambda_+^{ab}(x, y)$, only zero-modes corresponding to Θ^j_{odd} remain and the number of zero-modes is equal to $(M_a - M_b)/2 - 1$ for $(M_a - M_b) =$ even and $(M_a - M_b - 1)/2$ for $(M_a - M_b) =$ odd. The same holds true for λ_-^{ba}. Furthermore, when $M_a - M_b < 0$, the situation is the same except replacing $(M_a - M_b)$, λ_+^{ab} and λ_-^{ba} by $|M_a - M_b|$, λ_-^{ab} and λ_+^{ba}, respectively.

The 3-point couplings among modes corresponding to the wave functions, $\Theta^i_{\text{even,odd}}$, $\Theta^j_{\text{even,odd}}$ and $\Theta^k_{\text{even,odd}}$ are given by the overlap integral like (2.35). Note that

$$\int dy\ \Theta^i_{\text{even}}(y) \cdot \Theta^j_{\text{even}}(y) \cdot \Theta^k_{\text{odd}}(y) = \int dy\ \Theta^i_{\text{odd}}(y) \cdot \Theta^j_{\text{odd}}(y) \cdot \Theta^k_{\text{odd}}(y) = 0, \tag{3.8}$$

while $\int dy\ \Theta^i_{\text{even}}(y) \cdot \Theta^j_{\text{odd}}(y) \cdot \Theta^k_{\text{odd}}(y)$ and $\int dy\ \Theta^i_{\text{even}}(y) \cdot \Theta^j_{\text{even}}(y) \cdot \Theta^k_{\text{even}}(y)$ are nonvanishing.

3.3 U(N) gauge theory on magnetized orbifolds T^6/Z_2 and $T^6/(Z_2 \times Z_2')$

Here, we can extend the previous analysis on the two-dimensional orbifold T^2/Z_2 to the U(N) gauge theory on the six-dimensional orbifolds T^6/Z_2 and $T^6/(Z_2 \times Z_2')$. We consider two types of six-dimensional orbifolds, T^6/Z_2 and $T^6/(Z_2 \times Z_2')$. For the orbifold T^6/Z_2, the Z_2 twist acts on the six-dimensional coordinates y_m $(m = 4, \ldots, 9)$ as

$$y_m \to -y_m \quad (\text{for} \quad m = 4, 5, 6, 7), \qquad y_n \to y_n \quad (\text{for} \quad n = 8, 9). \tag{3.9}$$

In addition to this Z_2 action, we introduce another independent Z_2' action,

$$y_m \to -y_m \quad (\text{for} \quad m = 4, 5, 8, 9), \qquad y_n \to y_n \quad (\text{for} \quad n = 6, 7), \tag{3.10}$$

for the orbifold $T^6/(Z_2 \times Z_2')$. If magnetic flux is vanishing, we realize four-dimensional N=2 and N=1 supersymmetric gauge theories for the orbifolds, T^6/Z_2 and $T^6/(Z_2 \times Z_2')$, respectively.

Now, let us introduce the same magnetic flux as (2.26). The gauge group U(N) is broken as $\mathrm{U}(N) \to \prod_{a=1}^n \mathrm{U}(N_a)$ with $N = \sum_a N_a$. This magnetic flux is invariant under both Z_2 and Z_2' actions. Furthermore, we associate the Z_2 and Z_2' twists with the Z_2 and Z_2' actions in the gauge space as

$$\begin{aligned} A_\mu(x, -y_m, y_n) &= P A_\mu(x, -y_m, y_n) P^{-1}, \\ A_m(x, -y_m, y_n) &= -P A_m(x, -y_m, y_n) P^{-1}, \\ A_n(x, -y_m, y_n) &= P A_n(x, -y_m, y_n) P^{-1}, \end{aligned} \tag{3.11}$$

for $m = 4, 5, 6, 7$ and $n = 8, 9$, and

$$\begin{aligned} A_\mu(x, -y_m, y_n) &= P' A_\mu(x, -y_m, y_n) P'^{-1}, \\ A_m(x, -y_m, y_n) &= -P' A_m(x, -y_m, y_n) P'^{-1}, \\ A_n(x, -y_m, y_n) &= P' A_n(x, -y_m, y_n) P'^{-1}, \end{aligned} \tag{3.12}$$

JHEP09(2008)043

for $m = 4, 5, 8, 9$ and $n = 6, 7$. In general, these Z_2 boundary conditions break the gauge group $\prod_{a=1}^{n} \mathrm{U}(N_a)$ further. For simplicity, here we restrict to the Z_2 and Z_2' projections, which remain the gauge group $\prod_{a=1}^{n} \mathrm{U}(N_a)$ unbroken. That is, both the Z_2 and Z_2' actions are trivial for the unbroken gauge group.

Now, we study spinor fields. We focus on the $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$ block as (2.27) and use the same gauge as (2.28). We consider the spinor fields $\lambda^{aa}_{s_1,s_2,s_3}$, $\lambda^{ab}_{s_1,s_2,s_3}$, $\lambda^{ba}_{s_1,s_2,s_3}$ and $\lambda^{bb}_{s_1,s_2,s_3}$, where s_i denotes the chirality corresponding to the i-th T^2. Their Z_2 boundary conditions are given by

$$\lambda_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_2 P \lambda_{s_1,s_2,s_3}(x, y_m, y_n) P^{-1}, \tag{3.13}$$

with $m = 4, 5, 6, 7$ and $n = 8, 9$ for $\lambda^{aa}_{s_1,s_2,s_3}$, $\lambda^{ab}_{s_1,s_2,s_3}$, $\lambda^{ba}_{s_1,s_2,s_3}$ and $\lambda^{bb}_{s_1,s_2,s_3}$. Similarly, the Z_2' boundary conditions are given by

$$\lambda_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_3 P' \lambda_{s_1,s_2,s_3}(x, y_m, y_n) P'^{-1}, \tag{3.14}$$

with $m = 4, 5, 8, 9$ and $n = 6, 7$.

First, we study the gaugino fields $\lambda^{aa}_{s_1,s_2,s_3}$ and $\lambda^{bb}_{s_1,s_2,s_3}$ for the unbroken gauge group. Their zero-mode equations have no effect due to magnetic fluxes, but only the Z_2 and Z_2' orbifold twists play a role. Since the Z_2 and Z_2' twists, P and P', are trivial for the unbroken gauge sector, the boundary conditions are given by

$$\lambda^{aa(bb)}_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_2 \lambda^{aa(bb)}_{s_1,s_2,s_3}(x, y_m, y_n) \qquad \text{for} \quad Z_2, \tag{3.15}$$

with $m = 4, 5, 6, 7$ and $n = 8, 9$, and

$$\lambda^{aa(bb)}_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_3 \lambda^{aa(bb)}_{s_1,s_2,s_3}(x, y_m, y_n) \qquad \text{for} \quad Z_2', \tag{3.16}$$

with $m = 4, 5, 8, 9$ and $n = 6, 7$. Hence, zero modes of $\lambda^{aa(bb)}_{+,+,\pm}$ and $\lambda^{aa(bb)}_{-,-,\pm}$ survive on T^6/Z_2, that is, two kinds of gaugino fields with a fixed four-dimensional chirality. Furthermore, on $T^6/(Z_2 \times Z_2')$, zero modes of $\lambda^{aa(bb)}_{+,+,+}$ and $\lambda^{aa(bb)}_{-,-,-}$ survive, that is, a single sort of gaugino fields with a fixed four-dimensional chirality.

Next, let us study the bi-fundamental matter fields, $\lambda^{ab}_{s_1,s_2,s_3}$ and $\lambda^{ba}_{s_1,s_2,s_3}$. Without the Z_2 projection, they have zero-modes, whose number is $I_{ab} = I^1_{ab} I^2_{ab} I^3_{ab}$ and wave functions are given by $\Theta^{j_1}(y_4, y_5)\Theta^{j_2}(y_6, y_7)\Theta^{j_3}(y_8, y_9)$ ($j_i = 0, \ldots, (I^i_{ab} - 1)$). We assume that $I^i_{ab} > 0$ for $i = 1, 2, 3$. Then, the zero-modes correspond to $\lambda^{ab}_{+,+,+}$. On T^6/Z_2, some of them are projected out. Suppose that the Z_2 boundary condition is given by

$$\lambda^{ab}_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_2 \lambda^{ab}_{s_1,s_2,s_3}(x, y_m, y_n), \tag{3.17}$$

with $m = 4, 5, 6, 7$ and $n = 8, 9$. Then, surviving zero-modes correspond to $\Theta^{j_1}_{\text{even}}(y_4, y_5)\Theta^{j_2}_{\text{even}}(y_6, y_7)\Theta^{j_3}(y_8, y_9)$ and $\Theta^{j_1}_{\text{odd}}(y_4, y_5)\Theta^{j_2}_{\text{odd}}(y_6, y_7)\Theta^{j_3}(y_8, y_9)$. Further modes are projected out on $T^6/(Z_2 \times Z_2')$. Suppose that the Z_2' boundary condition is given by

$$\lambda^{ab}_{s_1,s_2,s_3}(x, -y_m, y_n) = s_1 s_3 \lambda^{ab}_{s_1,s_2,s_3}(x, y_m, y_n), \tag{3.18}$$

with $m = 4,5,8,9$ and $n = 6,7$. Then, the surviving modes through the $Z_2 \times Z_2'$ projection correspond to $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and $\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm odd}(y_6,y_7)\Theta^{j_3}_{\rm odd}(y_8,y_9)$. Similarly, we can analyze surviving zero-modes through the $Z_2 \times Z_2'$ projection in the models with different signs of I^i_{ab} and different $Z_2 \times Z_2'$ projections. It would be convenient to introduce the notation, $I^i_{ab({\rm even})}$ and $I^i_{ab({\rm odd})}$, such that $I^i_{ab({\rm even})}$ and $I^i_{ab({\rm odd})}$ denote the number of even and odd functions, $\Theta^j_{\rm even}$ and $\Theta^j_{\rm odd}$, respectively, among $|I^i_{ab}|$ functions Θ^j for the i-th T^2. Note that $I^i_{ab({\rm even})}, I^i_{ab({\rm odd})} \geq 0$ in the above definition, while I^i_{ab} can be negative.

3.4 $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$ models with three families

Here, we consider the $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$ models, which are derived from ten-dimensional $\mathrm{U}(N)$ super Yang-Mills theory on $R^{3,1} \times T^6/(Z_2 \times Z_2')$ with the same magnetic flux as in the previous subsection, e.g. $N_a = 4, N_b = 2$ and $N_c = 2$. Suppose that four-dimensional N=1 supersymmetry remains. In addition to the $\mathrm{U}(N_a)\times\mathrm{U}(N_b)\times\mathrm{U}(N_c)$ vector multiplets, the massless spectrum includes three types of bi-fundamental fields, $(N_a, \bar{N}_b)$, $(N_b, \bar{N}_c)$ and $(N_c, \bar{N}_a)$. As section 2.5, we assign $(N_a, \bar{N}_b)$ and $(\bar{N}_a, N_c)$ to left-handed and right-handed matter fields. In this case, $(N_b, \bar{N}_c)$ modes correspond to Higgs fields.

Now, we give explicit models. For simplicity, we restrict ourselves to models, which satisfy the condition (2.33). For example, we introduce the following magnetic flux,

$$\begin{aligned}
F_{45} &= 2\pi \begin{pmatrix} 0 \times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -3 \times \mathbf{1}_{N_b\times N_b} & \\ 0 & & -4 \times \mathbf{1}_{N_c\times N_c} \end{pmatrix}, \\
F_{67} &= 2\pi \begin{pmatrix} 0 \times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -4 \times \mathbf{1}_{N_b\times N_b} & \\ 0 & & -1 \times \mathbf{1}_{N_c\times N_c} \end{pmatrix}, \\
F_{89} &= 2\pi \begin{pmatrix} 0 \times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -1 \times \mathbf{1}_{N_b\times N_b} & \\ 0 & & 3 \times \mathbf{1}_{N_c\times N_c} \end{pmatrix}.
\end{aligned} \tag{3.19}$$

This magnetic flux breaks the gauge group $\mathrm{U}(N) \to \mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$, and satisfies the condition (2.33). In addition, we consider the orbifold projections, e.g.

$$P = P' = \begin{pmatrix} \mathbf{1}_{N_a\times N_a} & & 0 \\ & -\mathbf{1}_{N_b\times N_b} & \\ 0 & & \mathbf{1}_{N_c\times N_c} \end{pmatrix}, \tag{3.20}$$

which do not break $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$. A single sort of $\mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$ gaugino fields remain through the orbifold projection.

Now, let us study the spinor fields λ^{ab}. Their Dirac equations have the difference of magnetic fluxes, $I^i_{ab} = (3,4,1)$. Thus, their zero-modes correspond to $\lambda^{ab}_{+,+,+}$, which transform $\lambda^{ab}_{+,+,+}(x,-y_m,y_n) \to -\lambda^{ab}_{+,+,+}(x,y_m,y_n)$ for both Z_2 and Z_2' actions. In general, these boundary conditions are satisfied with both types of the wave functions

	$I^i_{\rm ef}$	chirality	wave function	the total number of zero-modes
λ^{ab}	$(3,4,1)$	$\lambda^{ab}_{+,+,+}$	$\Theta^{j_1}_{\rm odd}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm even}$	3
λ^{ca}	$(-4,-1,3)$	$\lambda^{ca}_{-,-,+}$	$\Theta^{j_1}_{\rm even}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm odd}$	3
λ^{bc}	$(1,-3,-4)$	$\lambda^{bc}_{+,-,-}$	$\Theta^{j_1}_{\rm even}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm even}$	6

Table 2: Three-family model.

$\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm odd}(y_6,y_7)\Theta^{j_3}_{\rm odd}(y_8,y_9)$. However, note that this model has $I^3_{ab}=1$ and $I^3_{ab({\rm odd})}=0$, that is, there is no zero-mode corresponding to $\Theta^{j_3}_{\rm odd}(y_8,y_9)$. Thus, the zero-modes correspond to only the wave functions $\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and the total number of zero-modes is given by the product of $I^1_{ab({\rm odd})}=1$, $I^2_{ab({\rm even})}=3$ and $I^3_{ab({\rm even})}=1$, that is, there are three zero-modes. The magnetic flux difference, $I^i_{ab}=(3,4,1)$, which would have twelve families of $(N_a,\bar{N}_b)$ without orbifolding. However, the orbifold projection reduces the family number from twelve to three. Similarly, we can analyze zero-modes for λ^{bc} and λ^{ca}. The result is shown in table 2. The second column shows magnetic fluxes, which appear in their Dirac equations, and the subscript ef denotes $ef=ab,ca$ and bc. The third and fourth columns show six-dimensional chiralities of zero-modes and their forms of wave functions. The fifth column shows the total number of zero-modes. This model has three families when we consider λ^{ab} and λ^{ca} as left-handed and right-handed matter fields. The scalar fields associated with λ^{bc} would correspond to Higgs fields. However, their Yukawa couplings are not allowed in this model, because of the periodicity (3.8).

We show another model with the following magnetic flux,

$$\begin{aligned}
F_{45} &= 2\pi\begin{pmatrix} 0\times\mathbf{1}_{N_a\times N_a} & & 0 \\ & -3\times\mathbf{1}_{N_b\times N_b} & \\ 0 & & \mathbf{1}_{N_c\times N_c}\end{pmatrix}, \\
F_{67} &= 2\pi\begin{pmatrix} 0\times\mathbf{1}_{N_a\times N_a} & & 0 \\ & -4\times\mathbf{1}_{N_b\times N_b} & \\ 0 & & -5\times\mathbf{1}_{N_c\times N_c}\end{pmatrix}, \\
F_{89} &= 2\pi\begin{pmatrix} 0\times\mathbf{1}_{N_a\times N_a} & & 0 \\ & -1\times\mathbf{1}_{N_b\times N_b} & \\ 0 & & -4\times\mathbf{1}_{N_c\times N_c}\end{pmatrix}.
\end{aligned} \tag{3.21}$$

This magnetic flux breaks the gauge group $\mathrm{U}(N)\to\mathrm{U}(N_a)\times\mathrm{U}(N_b)\times\mathrm{U}(N_c)$, and satisfies the condition (2.33). We consider the following orbifold projections,

$$P=\begin{pmatrix} -\mathbf{1}_{N_a\times N_a} & & 0 \\ & \mathbf{1}_{N_b\times N_b} & \\ 0 & & \mathbf{1}_{N_c\times N_c}\end{pmatrix},$$

	$I^i_{\rm ef}$	chirality	wave function	the total number of zero-modes
λ^{ab}	$(3,4,1)$	$\lambda^{ab}_{+,+,+}$	$\Theta^{j_1}_{\rm odd}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm even}$	3
λ^{ca}	$(1,-5,-4)$	$\lambda^{ca}_{+,-,-}$	$\Theta^{j_1}_{\rm even}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm odd}$	3
λ^{cb}	$(4,-1,-3)$	$\lambda^{cb}_{+,-,-}$	$\Theta^{j_1}_{\rm odd}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm odd}$	1

Table 3: Three-family model.

$$P' = \begin{pmatrix} \mathbf{1}_{N_a\times N_a} & & 0 \\ & -\mathbf{1}_{N_b\times N_b} & \\ 0 & & \mathbf{1}_{N_c\times N_c} \end{pmatrix}. \tag{3.22}$$

Then, we can analyze the zero-modes as the above. The result for bi-fundamental matter is shown in table 3. This model has three families of λ^{ab} and λ^{ca}. The scalar fields associated with λ^{bc} can couple with them. The three families of λ^{ab} and λ^{ca} are quasi-localized at points different from each other on the second T^2. Furthermore, one family of λ^{ab} and λ^{ca} is quasi-localized at the same point as the point, where λ^{bc} is quasi-localized. That could explain one family has a large Yukawa coupling with the Higgs fields, while the other families have smaller Yukawa couplings. However, the full form of Yukawa matrices is not completely realistic, because the up and down sectors of quarks have the same form of Yukawa matrices. We would study Yukawa matrices numerically elsewhere taking into account $\mathrm{SU}(2)_R$ breaking.

Similarly, we can construct other three-family models, which satisfy the condition (2.33). Also the model construction can be done in a similar way when we do not take into account the condition (2.33).

These two models are not completely realistic, but simple models to illustrate explicit model building. One of important features is that the family number is smaller than the magnetized torus models with the same magnetic fluxes and there are a variety of models with a fixed number of families, e.g. three-family models. Another important feature in generic model is that adjoint matter fields except gaugino fields are projected out by the orbifold projection on $T^6/(Z_2\times Z_2')$, although they remain on T^6/Z_2. Its implication from the viewpoint of model building is as follows. The above two models would correspond to the three-family Pati-Salam model when $N_a=4$, $N_b=2$ and $N_c=2$. In intersecting D-brane models, the Pati-Salam gauge group is broken by splitting D-branes to realize the breaking $\mathrm{U}(4)\to\mathrm{U}(3)\times\mathrm{U}(1)$ and $\mathrm{U}(2)\to\mathrm{U}(1)^2$ and such splitting correspond to the symmetry breaking by vacuum expectation values of adjoint scalar fields. However, we have no degree of freedom of adjoint scalar fields. One of simple ways to realize the standard gauge group in the above model building is that we start with $\mathrm{U}(6)\times\mathrm{U}(1)'_a\times\mathrm{U}(1)'_c$. We introduce the same magnetic fluxes as (3.19) and (3.21) for $N_a=3$, $N_b=2$ and $N_c=1$ in $\mathrm{U}(6)=\mathrm{U}(N_a+N_b+N_c)$. In addition, we introduce the same magnitude of magnetic fluxes in $\mathrm{U}(1)'_a$ and $\mathrm{U}(1)'_c$ as those in $\mathrm{U}(N_a)$ and $\mathrm{U}(N_c)$ blocks of $\mathrm{U}(N)$, respectively. Then we can obtain supersymmetic standard models with three families of quarks and leptons.

Alternatively, we have the degree of freedom to introduce any field on the orbifold fixed points. That is, we can break the Pati-Salam gauge group into the standard gauge group by vacuum expectation values of brane modes like the adjoint scalar field or a pair of the Higgs fields $(4, 1, 2)$ and $(\bar{4}, 1, 2)$. Another way to break the gauge symmetry is to change the orbifold projection such that P and P' break the gauge group further.

In addition to the above Higgs fields, one can introduce any mode on the orbifold fixed points. For example, all of three families may not be originated from bulk modes, but some of quarks and leptons are originated from such brane modes. That is, we have an interesting variety for model building. In appendix, we give two examples of models, whose family numbers of bulk modes are not equal to three. Furthermore, such brane modes can not couple with bulk modes, whose wave functions include $\Theta^j_{\rm odd}$ for the i-th T^2, because the wave function $\Theta^j_{\rm odd}$ vanishes on the fixed point. That is a new aspect in our model. In the usual orbifold models, bulk zero-modes correspond to even functions. Thus, they can couple with brane modes. However, in our model some of bulk modes can not couple with brane modes. This fact would be important for further model building.

4. Conclusion

We have studied D-dimensional N=1 super Yang-Mills theory on the orbifold background with non-vanishing magnetic fluxes, in particular spinor fields. Our models have a rich structure. Odd modes can have zero-modes and couplings are controlled by the orbifold periodicity of wave functions. We can derive flavor structures different from those in magnetized torus models. Thus, further study on model building would be interesting.

We have shown rather simple model building, although we could consider more generic class of magnetized orbifold models. We have more degrees of freedom of extensions for model building. One extension is to introduce brane modes as mentioned in section 3.4. In addition, we can introduce magnetic fluxes on orbifold fixed points, which would be independent of bulk magnetic fluxes and/or magnetic fluxes on different fixed points.[6]

We have started with D-dimensional N=1 super Yang-Mills theory. However, we can add hypermultiplets e.g. for D=6. Also we have considered six-dimensions and ten-dimensions, but we can consider eight-dimensional theory in a similar way. Moreover, we can extend our analysis to several combinations of branes, whose dimensions are different from each other like D9-D5 branes and other combinations.

We have restricted abelian magnetic fluxes, but in general non-Abelian magnetic fluxes are possible, i.e. the toron background. That reduces the rank of the gauge group. Furthermore, we can choose the orbifold projections, which break the gauge group further. Moreover, we have considered the factorizable orbifold and non-vanishing magnetic fluxes $F_{2m,2m+1}$ for $m = 2, 3, 4$. We could extend to non-factorizable orbifolds [23] and more generic form of magnetic fluxes.

Thus, we have various directions of extensions in generic magnetized orbifold models. Including these extensions, the structure of models would become much richer. Hence, further studies with these extensions are quite important.

[6] See e.g. [22].

JHEP09(2008)043

	I^i_{ef}	chirality	wave function	the total number of zero-modes
λ^{ab}	$(2,1,1)$	$\lambda^{ab}_{+,+,+}$	$\Theta^{j_1}_{\text{even}}\Theta^{j_2}_{\text{even}}\Theta^{j_3}_{\text{even}}$	2
λ^{ca}	$(2,1,1)$	$\lambda^{ca}_{+,+,+}$	$\Theta^{j_1}_{\text{even}}\Theta^{j_2}_{\text{even}}\Theta^{j_3}_{\text{even}}$	2
λ^{cb}	$(4,2,2)$	$\lambda^{cb}_{+,+,+}$	$\Theta^{j_1}_{\text{even}}\Theta^{j_2}_{\text{even}}\Theta^{j_3}_{\text{even}}$	12

Table 4: Two-family model from the bulk.

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A. Models

Here we give two examples of models, whose family numbers of bulk modes differ from three. That is, one model has two bulk families and the other has eighteen bulk families. We start with the ten-dimensional $\mathrm{U}(N)$ super Yang-Mills theory on the background $R^{3,1} \times T^6/(Z_2 \times Z_2')$. We consider the trivial orbifold projections $P = P' = 1$.

In the first model, we introduce the following magnetic flux,

$$\begin{aligned} F_{45} &= \begin{pmatrix} 0 \times \mathbf{1}_{N_a \times N_a} & & 0 \\ & -2 \times \mathbf{1}_{N_b \times N_b} & \\ 0 & & 2 \times \mathbf{1}_{N_c \times N_c} \end{pmatrix}, \\ F_{67} &= \begin{pmatrix} 0 \times \mathbf{1}_{N_a \times N_a} & & 0 \\ & -1 \times \mathbf{1}_{N_b \times N_b} & \\ 0 & & 1 \times \mathbf{1}_{N_c \times N_c} \end{pmatrix}, \\ F_{89} &= \begin{pmatrix} 0 \times \mathbf{1}_{N_a \times N_a} & & 0 \\ & -1 \times \mathbf{1}_{N_b \times N_b} & \\ 0 & & 1 \times \mathbf{1}_{N_c \times N_c} \end{pmatrix}. \end{aligned} \tag{A.1}$$

This magnetic flux satisfies the condition (2.33) and breaks the gauge group $\mathrm{U}(N) \to \mathrm{U}(N_a) \times \mathrm{U}(N_b) \times \mathrm{U}(N_c)$, although the orbifold projections are trivial $P = P' = 1$. Then, we can analyze the zero-modes as section 3.4. The result is shown in table 4. This model has two bulk families, when we consider λ^{ab} and λ^{ca} as left-handed and right-handed matter fields. This flavor number is not realistic. However, in orbifold models it is possible to assume that one family appears on one of fixed points.

We give another example. We use the same orbifold projections, i.e. $P = P' = 1$. We

	$I^i_{\rm ef}$	No. of zero-modes $\Theta^{j_1}_{\rm even}\Theta^{j_2}_{\rm even}\Theta^{j_3}_{\rm even}$	No. of zero-modes $\Theta^{j_1}_{\rm odd}\Theta^{j_2}_{\rm odd}\Theta^{j_3}_{\rm odd}$	the total number of zero-modes
λ^{ab}	$(6,3,3)$	16	2	18
λ^{ca}	$(6,3,3)$	16	2	18
λ^{cb}	$(12,6,6)$	112	20	132

Table 5: Eighteen-family model from the bulk.

introduce the following magnetic flux,

$$\begin{aligned} F_{45} &= \begin{pmatrix} 0\times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -6\times \mathbf{1}_{N_b\times N_b} & \\ 0 & & 6\times \mathbf{1}_{N_c\times N_c} \end{pmatrix}, \\ F_{67} &= \begin{pmatrix} 0\times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -3\times \mathbf{1}_{N_b\times N_b} & \\ 0 & & 3\times \mathbf{1}_{N_c\times N_c} \end{pmatrix}, \\ F_{89} &= \begin{pmatrix} 0\times \mathbf{1}_{N_a\times N_a} & & 0 \\ & -3\times \mathbf{1}_{N_b\times N_b} & \\ 0 & & 3\times \mathbf{1}_{N_c\times N_c} \end{pmatrix}. \end{aligned} \tag{A.2}$$

We study the spinor fields λ^{ab}, in whose Dirac equations the difference of magnetic fluxes $I^i_{ab} = (6,3,3)$ appears. Their zero-modes correspond to $\lambda^{ab}_{+,+,+}$, which transform $\lambda^{ab}_{+,+,+}(x,y_m,y_n) \to \lambda^{ab}_{+,+,+}(x,-y_m,y_n)$ for both Z_2 and Z_2' actions. These boundary conditions are satisfied with the wave functions $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and $\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm odd}(y_6,y_7)\Theta^{j_3}_{\rm odd}(y_8,y_9)$. The number of zero-modes corresponding to the former wave functions is given by the product of $I^1_{ab({\rm even})} = 4$, $I^2_{ab({\rm even})} = 2$ and $I^3_{ab({\rm even})} = 2$, while the zero-mode number corresponding to the latter is given by the product of $I^1_{ab({\rm odd})} = 2$, $I^2_{ab({\rm odd})} = 1$ and $I^3_{ab({\rm odd})} = 1$. Thus, the total number of λ^{ab} zero-modes is equal to $18(=16+2)$. Similarly, we can analyze zero-modes for λ^{bc} and λ^{ca}. The result is shown in table 5. For these zero-modes, only two forms of wave functions are allowed, that is, one is $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and the other is $\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm odd}(y_6,y_7)\Theta^{j_3}_{\rm odd}(y_8,y_9)$. The numbers of zero-modes corresponding to the former and latter are shown in the third and fourth columns. Six-dimensional chirality of all zero-modes correspond to $\lambda_{+,+,+}$ and they are omitted in the table.

This model has 18 families. It seems that this family number is too large. However, we can reduce the light family number if we assume anti-families of $(\bar{N}_a, N_b)$ and $(N_a,\bar{N}_c)$ matter fields on fixed points and their mass terms with the above families of matter fields. Such mass terms are possible for zero-modes corresponding to $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$. Thus, when we assume n anti-families, the number of light families reduces to $(18-n)$. This type of models has an interesting aspect, that is, some families of matter fields correspond to $\Theta^{j_1}_{\rm even}(y_4,y_5)\Theta^{j_2}_{\rm even}(y_6,y_7)\Theta^{j_3}_{\rm even}(y_8,y_9)$ and other families of matter fields correspond to $\Theta^{j_1}_{\rm odd}(y_4,y_5)\Theta^{j_2}_{\rm odd}(y_6,y_7)\Theta^{j_3}_{\rm odd}(y_8,y_9)$. In

JHEP09(2008)043

general, other combinations of wave functions can appear in zero-modes of matter fields. Such asymmetry appears in this type of models. Thus, their flavor structure is rich.

References

[1] N.S. Manton, *Fermions and parity violation in dimensional reduction schemes*, *Nucl. Phys.* **B 193** (1981) 502;
G. Chapline and R. Slansky, *Dimensional reduction and flavor chirality*, *Nucl. Phys.* **B 209** (1982) 461;
S. Randjbar-Daemi, A. Salam and J.A. Strathdee, *Spontaneous compactification in six-dimensional Einstein-Maxwell theory*, *Nucl. Phys.* **B 214** (1983) 491;
C. Wetterich, *Dimensional reduction of Weyl, Majorana and Majorana-Weyl spinors*, *Nucl. Phys.* **B 222** (1983) 20;
P.H. Frampton and K. Yamamoto, *Unitary flavor unification through higher dimensions*, *Phys. Rev. Lett.* **52** (1984) 2016;
P.H. Frampton and T.W. Kephart, *Left-right asymmetry from the eight sphere*, *Phys. Rev. Lett.* **53** (1984) 867;
K. Pilch and A.N. Schellekens, *Symmetric vacua of higher dimensional Einstein Yang-Mills theories*, *Nucl. Phys.* **B 256** (1985) 109.

[2] E. Witten, *Some properties of $O(32)$ superstrings*, *Phys. Lett.* **B 149** (1984) 351.

[3] C. Bachas, *A way to break supersymmetry*, `hep-th/9503030`.

[4] M. Berkooz, M.R. Douglas and R.G. Leigh, *Branes intersecting at angles*, *Nucl. Phys.* **B 480** (1996) 265 [`hep-th/9606139`].

[5] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, *JHEP* **10** (2000) 006 [`hep-th/0007024`].

[6] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type-I strings on magnetised orbifolds and brane transmutation*, *Phys. Lett.* **B 489** (2000) 223 [`hep-th/0007090`].

[7] D. Cremades, L.E. Ibáñez and F. Marchesano, *Computing Yukawa couplings from magnetized extra dimensions*, *JHEP* **05** (2004) 079 [`hep-th/0404229`].

[8] J. Troost, *Constant field strengths on T^{2n}*, *Nucl. Phys.* **B 568** (2000) 180 [`hep-th/9909187`].

[9] G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, *$D = 4$ chiral string compactifications from intersecting branes*, *J. Math. Phys.* **42** (2001) 3103 [`hep-th/0011073`]; *Intersecting brane worlds*, *JHEP* **02** (2001) 047 [`hep-ph/0011132`].

[10] R. Blumenhagen, B. Körs and D. Lüst, *Type I strings with F- and B-flux*, *JHEP* **02** (2001) 030 [`hep-th/0012156`].

[11] M. Cvetič, G. Shiu and A.M. Uranga, *Three-family supersymmetric standard like models from intersecting brane worlds*, *Phys. Rev. Lett.* **87** (2001) 201801 [`hep-th/0107143`]; *Chiral four-dimensional $N = 1$ supersymmetric type IIA orientifolds from intersecting $D6$-branes*, *Nucl. Phys.* **B 615** (2001) 3 [`hep-th/0107166`].

[12] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [`hep-th/0502005`];
R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, *Phys. Rept.* **445** (2007) 1 [`hep-th/0610327`].

[13] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, *Nucl. Phys.* **B 261** (1985) 678; *Strings on orbifolds. 2*, *Nucl. Phys.* **B 274** (1986) 285.

[14] G. 't Hooft, *A property of electric and magnetic flux in nonabelian gauge theories*, *Nucl. Phys.* **B 153** (1979) 141.

[15] J. Alfaro, A. Broncano, M.B. Gavela, S. Rigolin and M. Salvatori, *Phenomenology of symmetry breaking from extra dimensions*, *JHEP* **01** (2007) 005 [hep-ph/0606070];
D. Hernandez, S. Rigolin and M. Salvatori, *Symmetry breaking in six dimensional flux compactification scenarios*, arXiv:0712.1980.

[16] G. von Gersdorff, *A new class of rank breaking orbifolds*, *Nucl. Phys.* **B 793** (2008) 192 [arXiv:0705.2410].

[17] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory. Vol. 2: Loop amplitudes, anomalies and phenomenology*, Cambridge Monographs On Mathematical Physics, Cambridge University Press, Cambridge U.K. (1987).

[18] R. Blumenhagen, L. Görlich and T. Ott, *Supersymmetric intersecting branes on the type IIA $T^6/\mathbb{Z}_4$ orientifold*, *JHEP* **01** (2003) 021 [hep-th/0211059];
M. Cvetič and I. Papadimitriou, *More supersymmetric standard-like models from intersecting D6-branes on type IIA orientifolds*, *Phys. Rev.* **D 67** (2003) 126006 [hep-th/0303197];
M. Cvetič, T. Li and T. Liu, *Supersymmetric Pati-Salam models from intersecting D6-branes: a road to the standard model*, *Nucl. Phys.* **B 698** (2004) 163 [hep-th/0403061].

[19] T. Kobayashi, S. Raby and R.-J. Zhang, *Constructing 5D orbifold grand unified theories from heterotic strings*, *Phys. Lett.* **B 593** (2004) 262 [hep-ph/0403065]; *Searching for realistic 4D string models with a Pati-Salam symmetry: orbifold grand unified theories from heterotic string compactification on a Z_6 orbifold*, *Nucl. Phys.* **B 704** (2005) 3 [hep-ph/0409098].

[20] D. Cremades, L.E. Ibáñez and F. Marchesano, *Yukawa couplings in intersecting D-brane models*, *JHEP* **07** (2003) 038 [hep-th/0302105].

[21] T. Higaki, N. Kitazawa, T. Kobayashi and K.-J. Takahashi, *Flavor structure and coupling selection rule from intersecting D-branes*, *Phys. Rev.* **D 72** (2005) 086003 [hep-th/0504019].

[22] H.M. Lee, H.P. Nilles and M. Zucker, *Spontaneous localization of bulk fields: the six-dimensional case*, *Nucl. Phys.* **B 680** (2004) 177 [hep-th/0309195].

[23] S. Förste, T. Kobayashi, H. Ohki and K.-J. Takahashi, *Non-factorisable $Z_2 \times Z_2$ heterotic orbifold models and Yukawa couplings*, *JHEP* **03** (2007) 011 [hep-th/0612044].

JHEP09(2008)043